

Higher Mathematics - Homework 2

1. Simplify each expression

(a) $a^5 \times a^4$ (b) $n^{-12} \times n^9$ (c) $c^6 \times c$ (d) $d^{\frac{1}{2}} \times d^3$

(e) $3a^4 \times 5a^3$ (f) $4b^9 \times 2b^{-6}$ (g) $8c^8 \times 7c$ (h) $\frac{v^6}{v^2}$

i) $y^{19} \div y^{-5}$ (j) $\frac{k^8}{k}$ (k) $\frac{12c^5}{6c^3}$ (l) $\frac{48f^{10}}{6f^{-4}}$

(m) $30c^6 \div c^4$ (n) $(c^6)^5$ (o) $(y^7)^{-5}$ (p) $(6h^5)^3$

(q) $(2x^2)^5$ (r) $(xy)^5$ (s) $(x^2y^3)^4$ (t) $(h^3k^5)^{-8}$

2. Prove that the points A(-2, 1), B(-1, 0) and C(7, -8) are collinear.

3. The vertices of a triangle are A(2, 5), B(0, -5) and C(8, -1).

- (a) Find the equation of the altitude from A to BC.
- (b) Find the equation of the perpendicular bisector of AC.
- (c) Find the coordinates of the point T where these two lines meet.

4. A recurrence relation is defined as $U_{n+1} = aU_n + b$

- (a) List the first four terms of the sequence when $U_0 = 30$, $a = 0.8$ and $b = 4$.
- (b) Describe why this particular sequence has a limit.
- (c) Calculate the limit of this sequence.

5. A sequence is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

- (a) Given that $U_0 = 60$, $U_1 = 10$ and $U_2 = -15$, find k and c .
- (b) Find U_4 .

6. Find the equation of the line perpendicular to $y = 3x - 2$ which passes through (4, -3)

7. Differentiate the following expressions

a) $f(x) = x^6$ b) $f(x) = 4x^3$ c) $f(x) = -3x^5$ d) $f(x) = 2x^3 + 4x + 6$

e) $f(x) = \frac{1}{4x}$ f) $f(x) = \frac{4}{\sqrt{x}}$ g) $f(x) = 2\sqrt{x} + 3x$

11. Find the derivative of the following

a) $y = 4x^7 - 3x^{-2}$ b) $y = \frac{x+1}{x^2}$ c) $y = \frac{x^2-1}{x}$

12. Find the gradient of the curve $y = 4x^2 - 2x + 3$ at $x = 5$